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ABSTRACT

In an effort to present important research results in readable form for elementary and secondary teachers of mathematics, this publication reviews a model and ways of effectively teaching mathematics. The model of instruction proposed by Good and Grouws is based on teacher-centered whole class instruction. Perspectives and guidance are offered in the areas of: (1) making instructional improvement a priority (reviewing the characteristics of good teachers); (2) how the experts teach math (discussing the key aspects found in expert teaching); (3) effective mathematics teaching in the elementary grades (summarizing key instructional behaviors and teaching programs); (4) effective mathematics teaching in the junior high schools (describing instructional procedures adapted for use with junior high students); and (5) verbal problem solving (highlighting the importance of verbal problem solving). A listing of the literature on effective teaching of mathematics is also included. (ML)

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EFFECTIVE TEACHING OF MATHEMATICS

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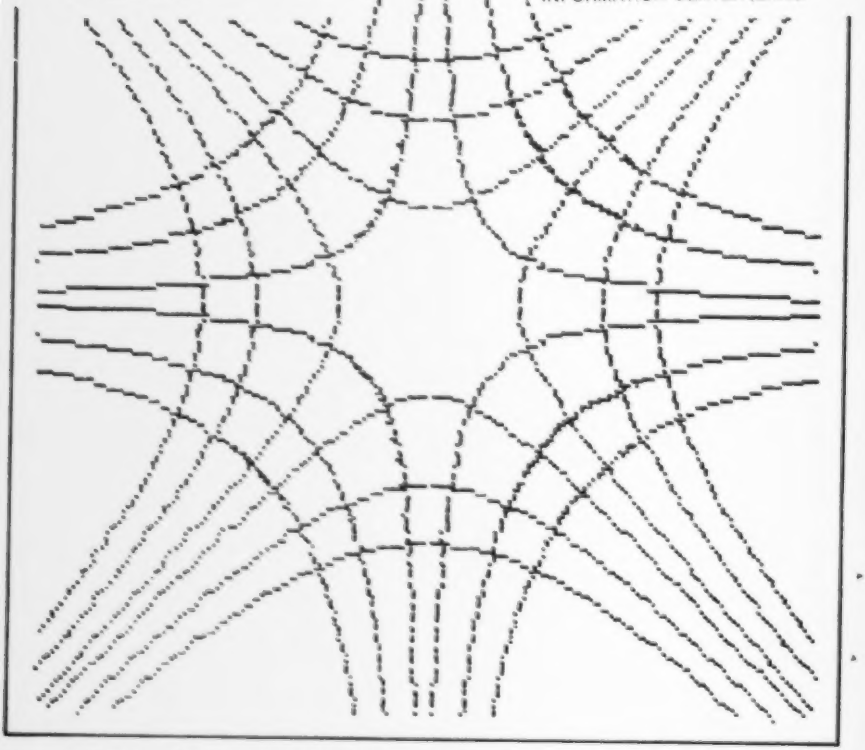

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EFFECTIVE TEACHING OF MATHEMATICS

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FOREWORD

Within the last ten years, educational research has yielded much new knowledge about how children learn mathematics and about the effective teaching of mathematics. Most Wisconsin mathematics teachers, who average 16 years of experience, have had little or no access to this recent research. Similarly, directors of instruction in local districts and undergraduate students preparing for careers as mathematics teachers have had little or no access to much of this material.

Professors Thomas Good and Douglas Grouws studied a large number of teachers over several years. They determined which teachers were effective, year after year, based on the achievement scores of their students. They then studied the behavior of the effective teachers to determine what characteristics might account for their effectiveness. The next step was to create a research study to determine under controlled conditions whether teachers using those identified behaviors (the experimental group) were more effective than teachers using their regular forms of instruction (the control group). Not only was there a significant difference in favor of the experimental group, but Good and Grouws found that they were able to teach teachers to use their model in only four hours. Thus, an extensive period of teacher training is not necessary to improve pupil performance.

The model of instruction proposed by Good and Grouws is teacher-centered whole class instruction. Successful teachers using other forms of instruction should not necessarily abandon those forms to take up the model described here. This model is recommended for teachers seeking alternatives to their present practice. Whole class instruction does not ignore the importance of meeting individual student's needs, but requires that teachers meet those needs within the context of whole class instruction.

The model, when implemented, produces an increase in academic learning time, the amount of time that students are *successfully engaged* in mathematics. Academic learning time is a variable which is known to be highly correlated with student achievement.

The publication of *Effective Teaching of Mathematics*, along with our previous publication of *Research Within Reach*: *Elementary School Mathematics*, is part of our continuing effort to bring important research results in readable form to the attention of elementary and secondary teachers of mathematics.

Donald L. Chambers
April, 1984

CHAPTER 1
MAKING INSTRUCTIONAL IMPROVEMENT A PRIORITY*

by

Donald L. Chambers

> No matter how teachers are identified as effective, whether by student achievement, by supervisors' recommendations, or by the testimony of students and classroom observers, it is evident that effective teachers of mathematics plan and behave differently from less effective teachers. More important, they do so in identifiable ways that can be learned by other mathematics teachers.
>
> —Mark Driscoll, "Effective Mathematics Teaching," *Research Within Reach: Secondary School Mathematics*

Efforts to improve school mathematics programs frequently focus on revision of the curriculum. While curriculum is certainly a fundamental variable in mathematics program improvement, instruction is a more critical variable and needs to receive more attention. According to the former United States Commissioner of Education Harold Howe II, "The improvement of pedagogy is the most likely source of a real solution to the shortcomings of America's schools."

Superintendents have an opportunity to make the improvement of mathematics instruction a priority for directors of instruction and principals. Superintendents can direct district funds, as well as Chapter 2 funds, to professional development activities for teachers of mathematics. Those activities can be designed to improve classroom instruction.

The characteristics of good teachers discussed here have been identified in the general literature on teaching—for example, *What Works: Research About Teaching and Learning* (U.S. Department of Education, 1986)—and are further supported by studies specific to the teaching and learning of mathematics. There is strong support for direct, teacher-centered whole group instruction, and the following recommendations can be implemented within that framework.

*Reprinted with permission of the American Association of School Administrators from *The School Administrator* 43(9) October 1986, pp. 18, 20. Donald L. Chambers is the mathematics consultant at the Wisconsin Department of Public Instruction.

Good Teachers Keep Students on Task

Good teachers make full use of their allocated time, keeping students successfully engaged in mathematics throughout the period. Good teachers begin instruction promptly at the beginning of the period and deal with homework assignments quickly and efficiently. Researchers Thomas Good, Douglas Grouws, and Howard Ebmeier (1983) found that many good teachers begin class with a brief statement reminding the students of the work they did the day before, move quickly through the homework (in five minutes or less), and then complete a few minutes of mental mathematics before moving on to the development of new material. Johnson (1982, 1986) has described a variety of strategies for getting class started promptly and dealing with the homework.

. . . Devote Half the Class Time to New Material

Good teachers devote at least half their class period to the development of new material. Many teachers fail to allow adequate time for this development. According to Welch (1978):

> In all math classes I visited, the sequence of activities was the same. First, answers were given for the previous day's assignment. The more difficult problems were worked by the teacher or a student at the chalkboard. *A brief explanation, sometimes none at all, was given of the new material*, and problems were assigned for the next day [emphasis added].

. . . Review Skills

Good teachers introduce the development with a review of previous skills and concepts that will be incorporated into the development. This recognizes that students are not blessed with total recall and will not remember all the ideas needed to develop the new material. By reviewing first, the continuity of the development of new material is not interrupted.

. . . Stress Concepts and Relationships

Good teachers develop an understanding of concepts and relationships rather than rely only on demonstrations of homework examples. Concept development is enhanced by the use of manipulative materials, pictures, and diagrams.

Good teachers keep students actively involved. Rather than rely on lectures that encourage passive learning, good teachers continuously involve students in activities and discussion.

... Emphasize Higher Order Thinking Skills

Good teachers emphasize higher order thinking skills and problem solving and model problem-solving behavior in class.

Good teachers emphasize estimation as an important and valuable skill in mathematics.

... Ask Process Questions

Good teachers ask more questions, especially process questions that require students to give an explanation as part of their answer. They allow adequate wait-time after a question and frequently ask follow-up questions after correct answers are given.

Good teachers assess student comprehension before leaving the development portion of the lesson. Student comprehension can be assessed by asking product questions (those requiring an answer but no explanation) as well as process questions. To assess comprehension, questioning should be directed to those students who are least likely to understand. Another strategy is to use controlled practice in which all students do one problem like the homework problems at their seats. The teacher can check understanding as students work, assigning a second problem if students perform well on the first or if more time is needed to monitor more students.

... Monitor Understanding

Good teachers do not move ahead to seatwork and homework until they have evidence that students will be successful. If only a few students are having trouble, their needs can be addressed during seatwork. Otherwise, seatwork should be deferred and lesson development should continue. They should reteach the next day, if necessary.

Good teachers use distributed practice to develop and maintain competence. Concepts learned during the lesson are practiced in class before the end of the period to consolidate learning. Those concepts are reviewed again as homework and are reviewed again the next day. Good teachers have regular weekly and monthly reviews and encourage students to maintain the competence they have developed by making every test a cumulative test.

... Use a Variety of Structures

Teachers philosophically committed to individualized or small group instruction and who are effective using that approach should not be compelled to switch to teacher-centered whole group instruction. There is some evidence that working in small, mixed-ability groups can enhance student achievement. One way to do this is to use teacher-centered whole group instruction for the development of the lesson and then use small mixed-ability groups for the seatwork. Robert Slavin and his colleagues (1984) have had some success with a program of Team-Assisted Individualization that involves students working with individualized instructional materials in teams.

Implementing only one or two of the suggestions above is unlikely to produce significant changes in the performance of pupils of marginal teachers. As Carolyn Evertson and others (1980) point out, "Effective teaching is not the mastery of a few key skills, it is the orchestration of a great many skills into a coherent system that meets the needs of the class."

Resources

Works cited in this article have been incorporated into the bibliography at the end of the book.

The author wishes to express his appreciation to Steven Leinwand, mathematics consultant, Connecticut Department of Education; Paul Hagerty, superintendent, Springfield Public Schools, Springfield, Missouri; and Richard Stolzmark, superintendent, Watertown Public Schools, Watertown, Wisconsin, for the valuable suggestions they made in the preparation of this article.

CHAPTER 2

HOW THE EXPERTS TEACH MATH*

by

Kay McKinney

Many students don't like math because they don't understand it. Kids who love math—and are very good at it—often have great teachers who can explain not only how to add and subtract but also the underlying concepts.

What makes a good math teacher? Several qualities, according to researchers at the Learning Research and Development Center (LRDC) at the University of Pittsburgh. LRDC is funded by the Education Department's Office of Educational Research and Improvement (OERI).

LRDC researchers spent six years studying seven expert math teachers who taught in elementary schools in very poor neighborhoods with "difficult" students or in economically depressed areas with high unemployment. They discovered that expert math teachers and novices proceed quite differently.

Wise Use of Time

Expert math teachers use almost all of the class time for math rather than for settling down and getting organized. It often takes beginning math teachers up to four minutes to pass out papers. In contrast, experts take less than 30 seconds. Novices can take as long as 15 minutes to correct the prior day's homework; experts take two or three minutes. Why the difference? Novice teachers often get distracted. For example, they stop to talk with students or make comments about their work that are unrelated to the task at hand.

Good teachers stretch the classroom hour by "stealing" time from blank spaces in the day. They play math games in the few minutes before recess when children are normally restless. Or they practice drills while the youngsters are lining up for lunch or the bus.

*This article is reprinted from the November 1986 "Research in Brief" by the U.S. Department of Education, Office of Educational Research and Improvement, Information Services Division.

They assign "useful" homework—math problems that let students practice what they have learned from that day's lessons. In contrast, less successful teachers may assign problems they have not yet explained to the students. Consequently, the homework either doesn't get done or the students do it wrong.

Expert teachers constantly drill students on what they have learned in class. This not only reinforces lessons but helps students gain confidence. Most expert teachers cover at least 40 problems a day through games, drills, or written work. Novice teachers, on the other hand, may cover only six or seven.

Organized Lessons

Good teachers know what they are going to teach and how they are going to teach it. They are unusually good at constructing a series of lessons that explains the content that needs to be learned. Their lessons are clear, accurate, and contain lots of examples and demonstrations.

They establish routines so that students know what is expected of them and what will take place during the lesson. Consequently, the teacher can get right to work teaching rather than explaining how the lesson will proceed. These teachers know what routines are needed to support a lesson. An expert teacher's daily routine might include reviewing homework, doing some drills, presenting a new concept, illustrating the concept at the blackboard, orally guiding the class through some sample problems, and finally, having the students do work at their desks.

Expert teachers use good, tightly organized demonstrations that clearly relate to the content being taught. They help their students understand what they are learning and why.

Although routines are important, they are not set in concrete. Good math teachers are flexible and willing to spend more or less time on each segment of the routine, depending on students' needs.

Students with expert teachers are rarely "lost" during a math lesson; they know what is happening, what is going to happen, and what they are supposed to do.

Know Their Content

Good math teachers know their subject. They know what children are going to find difficult and what materials or examples they can use to help them understand the content. Good teachers explain to students the problem, the solution, and when to use a specific process.

Students come away from a successful lesson knowing what they can do now that they could not do before and when to use the new process.

Expert math teachers set agendas to help guide them through a lesson. An agenda is a mental note pad teachers use to plot their moves, as well as those of their students, during a lesson. One item on an expert's agenda might include time to solve problems at the blackboard. This involves selecting children, sending them to the board, choosing problems, monitoring the students, and explaining corrections. Experts don't list all of these details in their agenda.

Novices, on the other hand, list a lot of moves in their mental notebooks. Consequently, they get so bogged down they often lose place during a lesson. Both they and their students end up confused.

Experts use concrete objects students can see and feel to help them learn the material better. These teachers know exactly which objects, such as blocks, can help students grasp a math concept. For instance, a teacher might demonstrate addition by having two piles of blocks—one with four blocks, another with three. To help children see what happens when they add, the teacher would push the two piles together.

Novices often are just one step ahead of the students and don't really know why certain things are done—"Why do we bring down a number in long division?," for example. Their explanations of new material often are fragmented and incomplete. Sometimes this lack of knowledge gets them into trouble because the material they use to explain something is more complicated than the concept they originally set out to explain.

These three aspects of expert teaching—wise use of time, organized lessons, and content knowledge—are intertwined. You can't have one without the others. For example, teachers with organized lessons but a lack of classroom management won't have time to teach their lessons.

Expert teachers know how to combine the three so that their students successfully learn math.

A copy of the complete set of studies, "Cognitive Skill of Teaching on Expertise in Math Teaching," is available from Gaea Leinhardt, University of Pittsburgh, Learning Research and Development Center, Pittsburgh, PA 15260. Please enclose a self-addressed label.

CHAPTER 3

EFFECTIVE MATHEMATICS TEACHING IN THE ELEMENTARY GRADES*

by

Thomas L. Good and Douglas A. Grouws
University of Missouri

I. INTRODUCTION

We believe it is possible to improve student performance in mathematics in important ways. By implementing the program which is outlined in the material that follows, we believe a significant difference in student performance will result.

We do not believe that any single teacher behavior will make a critical difference in student learning, but we do feel that several behaviors in combination can make a major impact. In the material that follows, we present a *system* of instruction that, if followed daily, will enhance student learning.

In general, we feel that the plan should be followed each day. However, we also realize that special circumstances will force you to modify the plan on occasion. Still, it is important that you follow the daily plan as frequently as you can.

For purposes of clarity, we will discuss each part of the teaching program separately. However, once again we want to emphasize that the program works when all parts are present. To maximize your opportunity for obtaining a clear picture of the instructional program, the program is summarized on the next page. The rationale for each part and how the pieces fit together will be discussed at a later point in the chapter.

*The development of this material was supported by the Grant NIE G 77 0003 (2526-2228) from the National Institute of Education. It is reprinted here with permission from the authors. Additional information and evaluation data from this project can be found in *Active Mathematics Teaching* by Good, Grouws, and Ebmeier, available from Longman Publishing.

SUMMARY OF KEY INSTRUCTIONAL BEHAVIORS*

Daily Review (First 10 minutes except Mondays)**

a) review the concepts and skills associated with the homework

b) collect and deal with homework assignments

c) ask several mental computation exercises

Development (20-25 minutes)

a) briefly focus on prerequisite skills and concepts

b) focus on meaning and promoting student understanding by using lively explanations, demonstrations, process explanations, illustrations, etc.

c) assess student comprehension

1) using process/product questions (active interaction)
2) using controlled practice

d) repeat and elaborate on the meaning portion as necessary

Seatwork (10-15 minutes)

a) provide uninterrupted successful practice

b) momentum—keep the ball rolling—get everyone involved, then sustain involvement

c) alerting—let students know their work will be checked at end of period

d) accountability—check the student's work

*Definitions of all terms and detailed teaching suggestions will follow.

**The suggested time allocations assume an instructional period of 50 minutes according to the recommendations of the Wisconsin Department of Public Instruction's *A Guide to Curriculum Planning in Mathematics*.

Homework Assignment (1-2 minutes)

a) assign on a regular basis at the end of each class except Fridays

b) should involve about 15 minutes of work to be done at home

c) should include one or two review problems

Special Reviews

a) Weekly Review/Maintenance

1) conduct during the first 20 minutes each Monday

2) focus on skills and concepts covered during the previous week

b) Monthly Review/Maintenance

1) conduct every fourth Monday

2) focus on skills and concepts covered since the last monthly review

II. DEVELOPMENT

Variable Description

The developmental portion of the mathematics period is that part of the lesson devoted to establishing comprehension of skills and concepts. It should be viewed as a continuum which runs from developing understanding to allowing for meaningful practice in a controlled setting. During all stages of the developmental portion of the lesson, both ends of the continuum may be present to some degree. However, in general, the comprehension emphasis with very little practice will come at the initial part of the lesson, then toward the middle of the lesson practice with process feedback from the teacher will become quite prominent, and finally in the latter portion of the lesson there will be controlled practice with meaningful explanations given as necessary.

The role of the teacher in the first part of the lesson, the comprehension phase, is to use instructional strategies that help students *understand* clearly the material being studied that day. In this portion of the lesson, emphasis is placed upon comprehension rather than rote memorization. Activities which often focus on comprehension include teacher explanations and demonstrations, and they may include use of manipulative materials to demonstrate processes and ideas, use of concrete examples in order to abstract common features, making comparisons and searching for patterns, and class discussions.

During the middle portion of the lesson, the number of questions posed to students may increase as the teacher begins to assess comprehension and provides them an opportunity to model processes already demonstrated and to verbalize the understanding they have developed. During this phase of the lesson, the teacher may decide that further explanations and demonstrations are necessary or decide that controlled practice is appropriate since students seem to understand what they are doing.

In the controlled practice phase of the lesson, the emphasis is on increasing proficiency; that is, increasing speed and accuracy. However, meaningful feedback is still given as necessary or requested.

Problem

Many problems arise in classes in which teachers give too little attention to development. Students exposed to such teaching frequently attempt to memorize rules for doing things and concentrate on mechanical skills. These rules have no meaning for the student (because developmental work was not done) and, thus, they are easily forgotten, especially when new sets of rules are "learned." When students do not understand what they are doing, each new problem causes them great difficulty. Often the comment, "We haven't done any of these before" is heard. When students learn without understanding, the ability to transfer skills to new situations is greatly reduced. Other negative results such as the inability to detect absurd answers and loss of self-confidence also occur. Thus, there are many compelling reasons to include a large measure of developmental work in mathematics lessons.

Teaching Suggestions

Initially, the teacher should focus briefly upon prerequisite skills that students may need for the lesson. Then the major aspect of the meaning portion of the development lesson occurs: active *demonstra-*

tion of the concept, idea, and/or skill that is being focused upon in the lesson, etc. Teachers need to demonstrate actively the process, so that students can comprehend the learning goal. You need to be cautious about moving too quickly into the assignment of problems and practice without providing students with an adequate conceptual orientation.

After the active demonstration and explanation by the teacher (and we recommend that ten minutes minimum be spent on this), the teacher should begin to *assess student comprehension*. There are two primary ways to do this. *First*, teachers may ask oral questions. In general, we recommend that teachers generally ask brief product-oriented questions. Product questions are questions that assess whether or not the student can produce the correct answer (see Appendix A for a complete description). Teachers can maintain an emphasis upon meaning by frequently providing process explanations *themselves* after students respond ("Yes, Tina, that's right *because* . . .").

The second way that teachers can assess student comprehension is by having students work practice problems. However, it is important to recognize that the role of a practice problem *in this stage of the lesson* is not to build up student speed and accuracy *per se*, but rather, the goal is to allow teachers to assess student comprehension. Hence, the assignment of problems in this stage should be limited to a single, brief problem followed by teacher assessment and explanation and then the provision of another brief problem assignment. In general, this stage of the lesson can be completed in three to five minutes.

If your questions or assigned problems reflect a moderate degree of student difficulty, then you should repeat the meaning portion of the lesson. If possible, use different examples; however, if this is not possible, verbatim repetition of the initial portion of the lesson is better than proceeding to controlled practice and seatwork when students are confused. Such a situation guarantees that students will practice errors.

If assessment of student comprehension is largely satisfactory, *then* teachers should proceed to the controlled practice portion of the development lesson. Now the teacher provides opportunity for students to develop increased speed, accuracy, and proficiency in completing problems of a specific type. However, the practice is still heavily controlled (unlike seatwork practice which will be discussed in the following section).

During controlled practice, teachers should assign only one or two problems at a time. *Students should not be asked to work longer than a minute without feedback about the correctness of their response.* The reason for this is that during the controlled part of the lesson, the

teacher is still trying to identify and correct any student misunderstanding. Too often many students are left to watch while a few students demonstrate a problem on the board. A great deal of practice time is lost this way and often the involvement of some students in the lesson (momentum) is lost as they become engaged in side conversations and distractions.

During controlled practice exercises, teacher accountability and alerting should be *immediate* and *continuous*. By alerting, we mean teacher behaviors that remind students that they should be doing work and that it will be checked. For example, if the teacher sends three to four students to the board to demonstrate the problem that students have just worked at their desks, the teacher might say, "Now the rest of you do these two new problems at your desks and I'll check them in a minute." Such teacher behavior maintains student *momentum*. Instead of watching classmates write on the board, they have their own work to do, and they are *alerted* to the fact that they will be held responsible for the work.

By *accountability* (more on this when we describe the seatwork portion of the lesson), we mean the actual checking of student responses. For example, while students put their work on the board, the teacher could look at the work of students who remain at their desks and check the problems that they were to have completed. Furthermore, the teacher can call on students to provide answers to practice problems, etc. Through such procedures, the teacher is able to assess when students are prepared to move to the seatwork portion of the lesson, when they have a longer block of time for *uninterrupted practice*. A final important characteristic of the controlled seatwork portion of the lesson is that the practice is done in the context of meaning (e.g., the teacher is frequently providing process explanations, "Yes, that's right *because* . . ."). Although the teacher is beginning to work for speed and accuracy, some attention is still being paid to students' understanding of the concepts, ideas, and skills that are being developed.

In summary, the development part of the lesson calls for the following teacher behavior:

1) Review briefly and/or identify prerequisite skills.

2) Focus upon the development of meaning and comprehension using active demonstration and teacher explanation.

3) Assess student comprehension (ask questions/work on supervised practice).

4) Repeat meaning portion of the lesson as necessary (using different examples and explanations if possible).

5) Provide practice opportunities for students.

   a) Practice should be short (one or two problems at a time).

   b) Students should be held responsible for assigned practice problems.

   c) Practice should be performed in a meaningful context (*teacher provides frequent process explanations*).

   d) When success rate is high, move students into seatwork portion of the lesson, when students have an opportunity for uninterrupted practice.

III. SEATWORK

Variable Description

Seatwork refers to practice work that students complete individually at their desks. Since seatwork practice follows the controlled practice part of the development lesson, students should know the purpose of assigned problems and how to do them when they begin to work. The role of seatwork practice is both important and easy to describe. Seatwork assignments allow students to practice, on their own, problems and principles that you have just actively taught. Seatwork provides students with an opportunity for *immediate* and *successful* practice. This practice experience allows students to achieve increased proficiency and to consolidate learning. New material or review work should *not* be assigned during the seatwork portion of the lesson.

Problem

Often a great deal of time is wasted when students work on problems individually. Indeed, research has consistently shown that students show less involvement (amount of time that students actually spend working on problems) during the seatwork portion of the lesson than during the active teaching portion of the lesson. Too often

teachers stop active supervision after they make the seatwork assignment. Two of the more common ways that teachers stop supervision are by doing desk work or by providing extended feedback to a single student (before all students are working on the task). Such behavior virtually guarantees that teachers cannot provide the type of supervision that students need if they are to begin to work productively. The first teaching task is to get students started on the seatwork. Often students do not use seatwork time productively simply because the teacher does not obtain their attention initially.

In addition to the problem of not "demanding" students to start work, some teachers create a problem by moving from the development portion of the lesson to seatwork with such abruptness that it is not surprising that students do not begin to work immediately (e.g., four students spring to the pencil sharpener, two students search for materials, and three students begin a private conversation). Momentum needs to be maintained throughout all stages of the lesson. When momentum is lost, students are apt to take a psychological break and once *momentum* (student attention and involvement) is lost, it is difficult to "recapture." Teachers who end the development portion of the lesson with a controlled practice segment have done much to ease the transition from the group lesson to the individual seatwork.

Teaching Suggestion

Given that the role of seatwork is to provide opportunity for successful practice, we recommend that about 10 to 15 minutes each day be allotted for seatwork. Ten to 15 minutes allows sufficient time for students to work enough problems to achieve increased proficiency but not so long as to bring about boredom, lack of task involvement, and the behavioral problems that soon follow when students are bored or frustrated. Frustration should be minimal to seatwork activity because the problems students are asked to do are a direct extension of the development part of the lesson. If practice time does not exceed 15 minutes, few students are likely to be bored.

The number of problems assigned should take most students only 15 minutes to complete. Hence, approximately 75 percent of your students should be able to complete the work within the allotted 15 minutes. In making the seatwork assignment, emphasis should be placed upon accurately working as many problems as possible within the allotted time. Low achievers who remain on task and do accurate work have done well and should know that they have done well. That is, the criterion to communicate to students is to keep working and to do as many problems accurately as they can.

To help optimize the effectiveness of seatwork, three general principles should be observed. The first principle, *momentum*, has already been discussed indirectly. By momentum, we mean keeping the ball rolling without any sharp break in teaching activity and in student involvement. Teachers can achieve momentum by ending the development portion of the lesson by working problems similar to the ones that students are asked to work individually and by starting students on individual work with a simple and direct statement. "We've worked problems 1 and 3. Now, individually, at your desk do problems 5 through 15. Work as many problems as you can, and we'll check our work in 15 minutes. Remember, doing the problem correctly is more important than speed." Following such a statement, *you should actively monitor all students*. Before providing feedback to individual students, make sure all students are engaged in the seatwork.

If some students do not begin working immediately, walk to their desks and if your physical presence doesn't initiate student work as it usually will, then quietly say something like "Frank, it's time to do the problems." After all students are working on the problems (the ball is rolling), you can then attend to the needs of individual students. In general, students should get immediate feedback and help when it is needed. Thus, it is usually reasonable to *allow* students to approach you when they have a question or problem. However, when presenting feedback to individual students, keep in mind the general principle of momentum. You have to provide feedback and conditions that allow most students to stay on task (keep working). Hence, it is not advisable to continue to provide lengthy feedback to an individual while several students are waiting for teacher feedback before they can continue to work.

Alerting is a second principle to observe during seatwork. Alerting behaviors tell students that they will be held accountable for their work. Often students engage in off-task behavior because they are not alerted to the fact that they will have their work checked at a specific point in time. If students are assigned seatwork that won't be checked until the following day (or when it is not checked at all), students are not likely to be highly engaged in seatwork. A statement like, "We'll check the work at the end of the period," alerts students to the fact that there is reason to engage in productive work immediately. A statement at the beginning of the seatwork is sufficient. Repeated statements are apt to *interfere* with students' work concentration. Public announcements should *not* occur during seatwork. Once you have students working, it doesn't make sense to distract them.

Accountability is the third principle to observe during seatwork. Alerting, as we noted, is a signal to students that their work will be

checked. Accountability is the actual checking of the work. It is important that your accountability efforts do not interrupt the seatwork behavior of students. During the controlled practice part of the lesson (see development section), accountability is *immediate*. However, during the seatwork portion of the lesson, students are to be working more independently, and those students capable of doing the work need time for *uninterrupted practice*. Public accountability needs to be *delayed* until the end of the lesson. A teacher's public questions during this stage of the lesson are very disruptive. For example, when the teacher asks a public question (e.g., "How many of you have done the first four problems?," "What's the answer to the second problem?," etc.), all students stop work, and once momentum is lost, some students will take much time before resuming their work. Furthermore, questions like "How many of you have finished the first four?" may make students anxious and distract them from task behavior if they have not worked the first four problems. Occasionally, you may need to stop seatwork practice to correct a common misunderstanding. In general, these errors should be corrected during the development (controlled practice) phase of the lesson. Public statements (except for necessary behavioral management) should be avoided. If most students are not ready for seatwork practice, then you should stay in the controlled practice part of the lesson. Such behavior will help students develop the following attitude toward seatwork: "I can do the problems, and now it is time for me to apply myself."

Perhaps the most direct and easiest way to hold students publicly accountable without disrupting seatwork is to call on individual students at the *end* of the lesson. Checking students' work at the end of the period also provides the teacher with a chance to spot any systematic mistakes that students are making and to correct those misunderstandings. Hence, when your students are assigned their homework, conditions should be set so that the homework provides for additional and relatively successful practice.

Specifically, we are asking you to *get student involvement* immediately after making a seatwork assignment. Continue to monitor and supervise all students until they are engaged in assigned work (*the first minute or two*). Early in the seatwork period (the first three to five minutes), be available for students when they need feedback. Toward the end of the seatwork period, try to get to the desks of some low achievers to see if they are making any systematic errors and to provide feedback as necessary. At the very end of the seatwork period, hold students accountable for their work by asking individual students to give the answer to a few of the assigned problems. This

checking of answers should be very rapid and you need only check three or four of the problems (check one or two problems at the first, in the middle, and at the end of the assigned work). If misunderstandings are corrected here, the homework should be a successful practice experience for most students.

When conducting the review of seatwork, it is generally advisable to call on low-achievement students to provide answers only to the first few problems assigned so as not to frustrate them for failure to complete all problems, but be sure to increase seatwork expectations for these students as the year progresses.

Finally, all seatwork should be collected. This helps encourage students to work productively because they know that they are held accountable for the work assigned during seatwork. Because of the way teachers have used seatwork in the past, many students have built up the expectation that seatwork is a time to relax and waste time. Taking up the seatwork will help students to adjust to the expectation that seatwork is a time to apply themselves and to see if they can do the type of problems which will be assigned as homework. Although there is no compelling reason to grade seatwork, it is important to examine the papers to see if students are using seatwork time appropriately. If a student's work is unduly incomplete, impossible to read, etc., it would be important to mention this to the student so that he or she knows that you care about seatwork performance.

After the seatwork is collected, the homework assignment is made. Delaying the assignment of homework helps to insure that students will do the work at a later point in time, hence, building distributed (repeated) practice into the mathematics program. Research has consistently shown the superiority of distributed practice over mass practice in helping students to master and retain new concepts and skills.

IV. HOMEWORK

Variable Description

Mathematics homework is written work done by students outside the mathematics class period. It is usually done at home; thus, it is distinctly different from seatwork which is done within mathematics class time.

Problem

The emphasis on homework in schools over the years has varied considerably. Unfortunately, homework has been missed frequently. Sometimes the assignments were so long that students became bored and careless when working the assigned problems. No doubt some students' dislike for mathematics is in part associated with these lengthy assignments. The instructional value of long homework assignments is very questionable. If students make errors on the first few problems of the assignment, then by the end of the assignment they may have become more proficient in making those errors!

Other situations in which homework has not been used to its full potential are plentiful. In some schools, homework is never given or so few problems are assigned that an excellent opportunity for distributed practice is wasted. Another undesirable situation occurs when homework is given primarily to please parents but without much attention to selecting problems and assignments that will foster progress toward important objectives. *But perhaps the most devastating misuse of homework is when children are assigned problems for which inadequate background has been developed in class.* While long assignments often lead to frustration, this latter situation always leads to frustration and negative attitudes toward the mathematics class.

Another situation which detracts from the value of homework assignments happens when the teacher fails to stress the importance and value of the problems assigned. This can be done directly by not commenting on the importance of assignments or indirectly by not scoring or collecting assignments.

In spite of these misuses of homework, homework can be an important part of mathematics learning if certain guidelines are followed. Research suggests that giving homework to students on a regular basis may increase achievement and improve attitudes toward mathematics. Short assignments have been found to be most effective, and some variety in the type of homework is helpful. Also, if a teacher gives importance to the homework through oral comments and by scoring papers regularly, then students frequently respond by completing their assignments with greater care.

Teaching Suggestion

Because of the important role that homework can play in improving student performance in mathematics, we suggest you do the following:

1) At the very end of the mathematics class period on Monday through Thursday, give a homework assignment which is due at the beginning of the class period the following day.

2) Each assignment should require about 15 minutes of outside class time. Within this time frame, assignments will probably average about eight problems per day depending on the kinds of problems being assigned. A typical assignment is shown in Appendix B.

3) The primary focus for an assignment should be on the major ideas discussed in class that day. Also, each assignment given on Tuesday and Wednesday should include one or two review problems from the current week's work.

4) Each assignment given on Thursday should be primarily devoted to review problems from the current week's work. In order for sufficient practice to be given on the material discussed on Thursday, this assignment will be a bit longer than assignments for other days and will probably take about 20 minutes for most students to complete.

5) Typically, each assignment should be scored (number correct) by another student. Papers should then be returned to their owners for brief examination. Finally, papers should be passed forward so that the scores can be recorded in the grade book.

6) The assignments given should be recorded daily in the Teacher's Log.

The short homework assignments complement seatwork by distributing practice over time without putting undue time pressure on students. Short assignments help hold student interest; adding variety to assignments is also helpful. This can be done by embedding the problems to be worked in different formats such as games, puzzles, codes, and so on. Appendix C illustrates this idea. Another component of variety might be to have students check their work. Multiplication problems can be checked by doing division, addition problems by doing subtraction, and so on. Variety can also be introduced by giving differentiated assignments. For example, some students could be given ten easy problems, while other students are given six problems of a more difficult nature.

The scoring and recording of grades on all homework assignments are designed to emphasize the importance of homework and to provide regular feedback to students and teachers regarding progress being made by each student. It is important to realize that there are a number of efficient ways to score homework other than the teacher's going through the papers individually. For instance, students can exchange papers or score their own papers. Either of these procedures is improved if students are expected to have their homework completed and ready to be scored at the very beginning of mathematics time. Efficiency is also improved if answers are prepared in advance by the teacher in written form (transparency, blackboard) and then shown to the students. Otherwise, the teacher may need to orally repeat each answer a large number of times.

Explanations and reteaching the homework must be somewhat limited if adequate time for discussion and practice of new material is to be available. This should not cause too much difficulty because most student difficulties and errors should have been remediated prior to the seatwork of the previous day.

A good strategy may be to quickly have children exchange and score papers, then have children indicate by raising their hands how many missed problems 1, 2, and so on. Then you can rapidly work the one or two problems that caused students the most difficulty. Since there are usually only a small number of homework problems to be checked and discussed, this part of the lesson should be easily completed in two minutes. Finally, note that any reteaching that is not completed can be done during the weekly review that is discussed in the next section.

In the rare event that the checking of homework reveals numerous student errors, you should reteach the previous day's lesson beginning with development, then controlled practice, then seatwork, and finally a homework assignment on the same material. Under these circumstances you should not try to cover new material due to the very limited amount of time available to develop the new ideas.

You should personally score the homework that is assigned on Thursdays. There are two reasons for this. First, the information gathered from this homework is to be used to structure the weekly review each Monday. Second, the focus of student scoring is of necessity on answers rather than kinds of errors being made. It is very important, however, that regular attention be given to the procedures and processes that students are using. This is especially true when they are making errors!

In connection with the scoring of Thursday's work, each student's paper should be analyzed for systematic error patterns. Systematic

error patterns refer to incorrect procedures which are consistently used on a wide range of problems. In two-digit multiplication problems, for example, a student might consistently forget to "carry" the tens digit from the initial multiplication of the units digits. According to recent research, such errors are much more common than was once realized and, thus, spending time examining homework with them in mind can be very helpful in remediating some students' difficulties with mathematics. Further examples of common computational error patterns can be found in Appendix D. Since the particular errors you find probably will not be associated with groups of students, the remediation of errors is usually best done on a one-to-one basis.

Homework is an important component of this program, and since both students and teachers devote a considerable amount of time to it, it is recommended that homework count at least 25 percent of each student's mathematics grade and that this information be communicated to them.

Parents are interested and should be informed about what is happening in school. Therefore, it is recommended that an explanation of the homework policy to be followed during the year be sent home to parents. A letter which could be duplicated and used for this purpose can be found in Appendix E.

Homework is explicitly related to each of the other components of instruction in a number of ways. With an increase in development time, it provides an opportunity to supplement the practice part of the lesson. It is structured such that practice is distributed over time and students have an opportunity to correct difficulties encountered in seatwork. The homework provides important information for structuring the specific details to be covered in the review component. It is also related to the pacing variable in that it allows some necessary work to be done outside of the time regularly scheduled for mathematics.

V. SPECIAL REVIEW/MAINTENANCE*

Variable Description

Children forget. It is imperative, therefore, that ideas be reviewed and skills maintained on a systematic basis in elementary school

*The review discussed here is in addition to the brief (one to four minute) daily review that we will discuss later.

mathematics. Reviewing ideas may involve the teacher stating and explaining properties, definitions, and generalizations and the students recalling the appropriate term or name. These roles occasionally may be reversed (where the teacher supplies a term and the students illustrate and explain), but the focus should generally be developmental in nature. That is, there should be a strong emphasis on meaning and comprehension. Similarly, skills need to be practiced with *regularity* in order that a *high level of proficiency be maintained*. The focus should be developmental in nature; comprehension again is an important component.

Problem

When discussing children's performance in mathematics, frequently the comment is made that many have not mastered the basic skills. From this it is concluded that teachers do not spend enough time teaching basic computation. But this conclusion often is not valid because the inability to perform may *not* be associated with the initial learning but rather the lack of maintenance. Newly learned material is particularly susceptible to being forgotten, but even material thought to be "mastered" is sometimes lost. For example, many fourth-grade teachers have had the experience in which a student seems to have mastered the basic multiplication facts, indeed, he or she can recall them with almost 100 percent accuracy but four weeks later seems to have forgotten a great number of them.

Teaching Suggestion

To minimize this problem and similar problems, we suggest that you incorporate review/maintenance sessions regularly into your mathematics instruction. Regularly in the sense that each Monday you have a Weekly Review/Maintenance session, and every fourth Monday you have a Cumulative Review/Maintenance session. The purpose of the two types of review sessions is to help students retain concepts and insights.

Weekly Review/Maintenance. The following things are necessary to do if the review/maintenance component is to be implemented effectively:

1) The first one-half of each Monday's mathematics period (roughly 25 minutes) should be devoted to review/maintenance.

2) The focus should be on the important skills and concepts covered during the previous week. The suggested order for covering these skills and ideas is:

   a) Those that are thought to be mastered and can be done very quickly

   b) Those that need additional development and practice as identified from the analysis of the Thursday homework assignment

   c) Those that need additional work (as identified during this review session)

Most of the important skills and concepts that should be reviewed can easily be identified by examining the homework assignments from the previous week. That is, these homework assignments deal with each important data or skill; thus, reviewing them will assist you in identifying important topics. It is of utmost importance that all major ideas covered during the week be reviewed. Reviewing ideas that students have "mastered" the previous week helps to guarantee that ideas will be retained. Areas in which some reteaching is definitely needed should be identified in advance by the teacher from an analysis of the Thursday homework assignment and handled during the second portion of this designated review segment.

There are many ways that this maintenance program can be successfully organized. One important attribute of any effective organizational scheme is active student involvement. In most teaching situations, it is important to avoid situations that involve only one student in checking problems because such a procedure is usually ineffective and boring to most children. This is especially true in a review situation in which students are already familiar with the problem. One scheme that we highly recommend (because it overcomes this difficulty) is one in which the teacher presents an idea or problem and then allows students to work individually at their desks until most arrive at an answer. Finally, answers are checked (children are held accountable), and someone explains or demonstrates how to arrive at the answer (in many cases by using the chalkboard at the front of the room).

Cumulative Review/Maintenance. This aspect of the review/maintenance program can best be implemented in the following way:

1) Every fourth Monday the entire period should be devoted to a cumulative review/maintenance session.

2) This review should encompass the work of the previous four weeks and, thus, replace the regular Monday review/maintenance session.

This session provides an opportunity to reteach ideas that have given difficulty over the past four weeks. It will be particularly useful to those students who have difficulty acquiring skills and ideas on initial exposure.

The interest in the value of this review session can be greatly enhanced by structuring it in an interesting format such as a game, contest, or quiz show.

Postscript

On occasion, it may be desirable to reschedule a review for a day other than Monday. For example, if by not reviewing on Monday you can complete a chapter or unit, by all means do this and simply conduct your review on Tuesday.

VI. MENTAL COMPUTATION

Variable Description

Mental computation is computation that is done without the aid of pencil and paper (or minicalculator). The process is done by the most powerful computer of all, the human brain. Mental processing is often vastly different than pencil and paper calculation. For example, in pencil and paper addition situations, the calculation always goes from *right* to *left*. The student asked to solve 41 + 12 on paper is going to move mechanically from right to left. However, in a mental activity (the teacher says, "What is 41 + 12?") the student may frequently move from *left* to *right*. First, the student does something to the tens column, then to the ones column, and then combines. We feel that the inclusion of some time for mental computation each day will help students to further develop their quantitative sense, to become more

flexible in thinking and in approaching problem-solving situations. Furthermore, such activities help students to detect absurd answers (e.g., when checking their written computation) and make estimations that are frequently needed in daily activities.

Problem

The attention given to mental computation and mental problem solving has largely disappeared from the modern mathematics curriculum. At one point in time much emphasis was given to mental problem solving. This de-emphasis has occurred despite some research evidence which suggests that mental practice on a regular basis appears to be related to large increases in student achievement. If students are not given some work in mental computation, then they are missing a very important way to check their work (other than the time-consuming and inefficient process of completely redoing the work).

Teaching Suggestion

We would like for you to include three to five minutes on mental computation activities each day at the beginning of the lesson; the predevelopment part of the lesson will be described later. Ideally, the material presented for mental resolution would be related to the content of the material being studied. During the study of subtraction, mental computation activities should focus on subtraction. However, some units that you study in the year will not lend themselves to this form of mental processing. During such a unit (e.g., geometry) it would be useful to rotate on a daily basis with the following types of mental computation activities: addition, subtraction, multiplication, division, and verbal problems.

The following examples will give you some ideas about the kinds of problems you may present to your students. Some of the examples here may be too easy or too difficult for your students. You should try to use problems which are challenging yet accessible to most students. It is a good idea to discuss how a problem might be solved before students are asked to give solutions.

For example, for a problem like 6×12 you might suggest thinking as follows: "6 times 12, that's 6 times 10 plus 6 times 2, that's $60 + 12$, 72." Then begin giving students problems one at a time to solve like 8×12, 6×15, and so on. It is worthwhile to mention to the students that there are *many ways to solve problems mentally* and the way you show is but one way. *Children should be encouraged to discuss their mental computation procedures.*

Further illustrations of the kinds of problems which are appropriate are given below. You should generate other types of mental computation exercises for your students as well.

Addition

1) 75 + 77 = _____
Think: 77 = 70 + 7. First add 70 to 75 (145) and then add 7 to that sum (152).
or:
Rename 77 as 70 + 7 and 75 as 70 + 5. Add the tens (140), add the ones (12), then find the total of the sums (152).

2) 97 + 8 = _____
Think: How much do I add to 97 to get 100? The answer is 3. Since 8 = 3 + 5, first add 3 to 97, and then add 5 to the sum.

3) 243 + 104 = _____
Think: 104 = 100 + 4. First add 100 to 243 and then add 4 to the sum.

4) 125 + 49 = _____
Think: 49 is 1 less than 50. Since 125 + 50 = 175, 125 + 49 = 174.

Subtraction

1) 125 - 61 = _____
Think: 61 = 60 + 1. First subtract 60 from 125, and then subtract 1 from the difference.

2) 105 - 8 = _____
Think: First subtract enough from 105 to get 100: 105 - 5 = 100. Since 8 = 5 + 3, subtract 3 more: 100 - 3 = 97.

3) 425 - 97 = _____
Think: 97 = 100 - 3. First subtract 100 from 425, and then add 3 to the difference. 425 subtract 100 is 325, add 3 is 328.

Multiplication

1) $20 \times 36 =$ _____
Think: $20 = 2 \times 10$. Ten times 36 is 360, and $2 \times 360 = 720$.
or: $2 \times 36 = 72$, so $20 \times 36 = 720$.
or: 20×36 that's the same as $(\frac{1}{2} \times 20) \times (2 \times 36)$, or $10 \times 72 = 720$.

2) $4 \times 17 \times 25 =$ _____
Think: Since the product of 4 and 25 is 100, these numbers are multiplied first. Then 100 is multiplied times 17.

3) $32 \times 50 =$ _____
Think: The product is unchanged if I double one factor and half the other factor. Thus, 32×50 is the same as 16×100, or 1,600.

4) $4 \times 53 =$ _____
Think: $53 = 50 + 3$. Four times 50 is 200. Four times 3 is 12. So to find 4×53 add $200 + 12$.

Division

1) $84 \div 4 =$ _____
Think: $84 = 80 + 4$. 80 divided by 4 is 20 and $4 \div 4$ is 1, so $84 \div 4$ is $20 + 1$, or 21.

2) $396 \div 4 =$ _____
Think: $396 = 400 - 4$. Since $400 \div 4 = 100$ and $4 \div 4 = 1$, the quotient is $100 - 1$ or 99.

3) $250 \div 50 =$ _____
Think: $250 \div 50$ is the same as $500 \div 100$ which is 5.

Verbal Problems

1) Mr. Thomas has a debt of \$120. If he pays \$70 of it, how large of a debt will he have left?
Think: I need to find $120 - 70 =$ _____
$12 - 7 = 5$, so $120 - 70 = 50$;
50 is the answer.

VII. INSTRUCTIONAL PACE

Variable Description

Instructional pace refers to rate. It may be thought of in terms of how quickly a class is moved through a given curriculum or in terms of how rapidly students are presented with particular topics. The pace associated with different teachers varies. Some teachers move through the curriculum faster than others.

Problem

Instructional pace may inhibit learning in several ways. At one extreme is the situation in which a teacher moves through the curriculum too quickly for learning to take place. At the other extreme is the teacher who plods along so slowly that many of the students are bored. Furthermore, some teachers, because of their slow pace, find themselves forced to cover so much material at the end of the year that they do not have time to build in the distributed practice which is essential if students are to retain the material. Research suggests that for most teachers efficiency could be improved if they increased their pace slightly. That is, there seems to be more of a tendency to procrastinate than to move forward. If the suggestions presented earlier in the chapter are implemented in your teaching program, the important element of review and distributed practice should be fulfilled, and you will probably be able to pick up the pace.

Teaching Suggestion

For this variable we suggest that you carefully consider your teaching behavior with respect to the instructional pace you set. Many of you will find that you can increase the pace somewhat, and we suggest you attempt to do so.

The instructional strategies suggested in this chapter are such that if you speed up a bit too much, then you can resolve problems that arise through your regularly scheduled review/maintenance sessions.

VIII. STARTING AND ENDING THE LESSON

We have now discussed the major parts of the mathematics instructional program. Two aspects that we have not discussed explicitly are the start and end of the lesson.

The beginning portion of the lesson (*Predevelopment*) will have three parts: (1) a brief review, (2) the checking of homework, and (3) some mental computation exercises. We suggest that all three of these activities be done within the first eight minutes of the class period. This may be difficult for teachers who slowly ease into the lesson, but it has been commonly observed that time is frequently used inefficiently at the beginning of a lesson.

The review of the previous day's lesson should begin with a *brief* summary by the teacher. Providing several sentences that briefly and concisely remind students of what they did and why and demonstrate how to solve a single problem are usually sufficient. Next comes the checking of homework. This should proceed very quickly once students learn that when the mathematics period begins they are to have their homework on top of their desks ready for checking. Initially, it may take some time to establish this routine, but once the routine is established it should take only a couple of minutes to check homework.

The third activity, mental computation, plays two roles in the lesson structure. First, it is an important activity *per se* (see earlier section). Second, these activities can provide a smooth transition for getting students engaged in thinking about mathematics prior to the point at which the teacher begins a new development lesson.

The *ending* of the lesson is a very simple procedure. After allowing students a period of time for uninterrupted practice, the teacher briefly checks pupils' work on a few problems (may call on students, ask students who got problems correct to raise their hands, etc.). This accountability procedure encourages students to apply themselves during seatwork and allows an additional opportunity to clear up misunderstanding. After checking some of the seatwork, the teacher ends the mathematics lesson by assigning the homework problems.

The predevelopment phase of the lesson should take roughly eight minutes. The exact distribution of time on *review*, *homework*, and *mental computations* depends upon a variety of conditions (e.g., moderate difficulty with homework vs. no difficulty), and you should use your judgment. In general, we think the following situation will be most applicable: one to two minutes on review; three to four minutes checking homework; and three to four minutes on mental computation.

IX. SUMMARY AND INTEGRATION

We have suggested you do several things to improve student performance in mathematics. In the first part of this chapter we emphasized that we didn't feel that changing one or two teacher behaviors would make much difference in student performance. We feel that the systematic application of all the behaviors discussed in this treatment program can make an important difference in student learning. The purpose of this last section is to briefly review the teaching suggestions we have made and to show how each of the pieces fits together into a total program.

The predevelopment portion of the lesson begins with a brief summary and a review of the previous lesson. The review (including the checking of homework) is designed to help students maintain conceptual and skill proficiency with material that has already been presented to them. Mental computation activities follow and provide an interesting bridge for moving into the new lesson.

Next comes the development part of the lesson which is designed to help students understand the new material. *Active* teaching helps the student comprehend what he or she is learning. Too often students work on problems without a clear understanding of what they are doing and the reasons for doing it. Under such conditions, learning for most students will be filled with errors, frustration, and poor retention. If student learning is to be optimal, students must have a clear picture of what they are learning; the development phase of the lesson is designed to accomplish this understanding.

The controlled practice that occurs toward the end of the development portion of the lesson is designed to see if students are ready to begin seatwork. It simply doesn't make sense to assign seatwork to students when they are not ready for it . . . practicing errors and a frustrating experience guarantees that student interest and performance in mathematics will decline. The controlled practice part of the lesson provides a decision point for the teacher. If students generally understand the process and are able to work problems correctly, then the teacher can proceed to the seatwork portion of the lesson. If student performance on problems is relatively poor, then the development must be *retaught*. If students are ready to do practice work, it is foolish to delay them; similarly, if students are not ready to do development work, it is foolish to push them into it. The controlled practice part of the lesson allows the teacher to decide if it is more profitable to move to seatwork or to reteach the development portion of the lesson.

Hence, when teachers move to the seatwork portion of the lesson, students should be ready to work on their own and practice should be relatively error free. Seatwork provides an opportunity for students to practice successfully the ideas and concepts presented to them during the development portion of the lesson. If teachers consistently present an active development lesson and carefully monitor student performance during the controlled portion of the lesson, then student seatwork will be a profitable exercise in *successful practice*.

The seatwork part of the lesson allows students to organize their own understanding of concepts (depend less upon the teacher) and to practice skills without interruption. The seatwork part of the lesson also allows the teacher to deal with those students who have some difficulty and to *correct their problems before they attempt to do homework*. If teachers actively monitor student behavior when seatwork is assigned and if they quickly engage them in task behavior and maintain that involvement with appropriate accountability and alerting techniques, then the essential conditions have been created for successful practice.

Homework is a logical extension of the sequence we have discussed. During the mathematics lesson, students learn in a meaningful setting. During seatwork, students have a chance to practice and deal with material they understand. The homework assignment provides an additional practice opportunity to further skill development and understanding.

The above aspects of the mathematics lesson combine to give students: (1) a clear understanding of what they are learning; (2) controlled practice and reteaching as necessary to reinforce the original concepts and skills; (3) seatwork practice to increase accuracy and speed; and (4) homework assignments which allow successful practice on mastered material (distributed practice which is essential to retention).

To maintain skills, it is important to build in some review. Skills not practiced and conceptual insights not reviewed from time to time tend to disappear. Even mature adults forget material and forget it rapidly. For this reason, we have suggested you provide for *review* of material presented the previous week *each Monday* and provide a *comprehensive review* every *fourth Monday*. Such procedures will help students to consolidate and retain their learning. Finally, we have suggested that the systematic presentation of mathematics material may facilitate student learning (i.e., initial acquisition) such that you can pick up the pace a bit, and we encourage you to do so if you can.

Finally, when many students experience trouble, the development portion of the lesson should be repeated, and students should never be asked to do homework until they are ready to do it successfully.

The plan described above is summarized in the Lesson Time Tables that follow. These tables outline the sequence and length of each lesson component in order to provide a general picture of the mathematics lesson that we are suggesting you teach.

LESSON TIME TABLE (WEEKS 1, 2, 3)

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
Weekly Review (25 minutes)	Homework Review Mental Computation (10 minutes)	Homework Review Mental Computation (10 minutes)	Homework Review Mental Computation (10 minutes)	Homework Review Mental Computation (10 minutes)
Development (10 minutes)	Development (20-25 minutes)	Development (20-25 minutes)	Development (20-25 minutes)	Development (20-25 minutes)
Seatwork (10 minutes)	Seatwork (10-15 minutes)	Seatwork (10-15 minutes)	Seatwork (10-15 minutes)	Seatwork (10-15 minutes)
Lesson Conclusion and Homework Assignment (2 minutes maximum)	Lesson Conclusion and Homework Assignment (2 minutes maximum)	Lesson Conclusion and Homework Assignment (2 minutes maximum)	Lesson Conclusion and Homework Assignment (2 minutes maximum)	Lesson Conclusion (2 minutes maximum)

LESSON TIME TABLE (WEEK 4)

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
Monthly Review (50 minutes)	Review Mental Computation (10 minutes)	Homework Review Mental Computation (10 minutes)	Homework Review Mental Computation (10 minutes)	Homework Review Mental Computation (10 minutes)
	Development (20-25 minutes)	Development (20-25 minutes)	Development (20-25 minutes)	Development (20-25 minutes)
	Seatwork (10-15 minutes)	Seatwork (10-15 minutes)	Seatwork (10-15 minutes)	Seatwork (10-15 minutes)
	Lesson Conclusion and Homework Assignment (2 minutes maximum)	Lesson Conclusion and Homework Assignment (2 minutes maximum)	Lesson Conclusion and Homework Assignment (2 minutes maximum)	Lesson Conclusion (2 minutes maximum)

APPENDIX A

PROCESS/PRODUCT QUESTIONS

Variable Description

Process questions ask the student to explain something in a way that requires him or her to integrate facts to show knowledge of interrelationships. Process questions often begin with *why* or *how* and can't be answered with one word. Many process questions require the student to specify the cognitive and/or behavioral steps that must be gone through in order to solve a problem or come up with an answer. Two examples of process questions follow. "If a man bought three tickets for $2.85 and two tickets for $2.15 and if we wanted to know the average cost per ticket, *how* could we get the answer?" Similarly, if the teacher asks "There are 60 minutes in an hour. How can we find out how many minutes in ¼ hour?," she or he is asking a process question. The student is asked to explain a process and to verbalize understanding ("We can always find ¼ of anything by dividing by 4 . . .").

Product questions only require a knowledge of a specific fact and can often be answered with a single word or by providing a number (answer to a problem). Product questions often begin with the words who, what, when, where, how much, how many, etc. A *written* example of a product question would be 7 + 3 = _____? An *oral* product question would be "zero times seven equals how much?"

Product questions can be transformed into process questions by asking for an explanation rather than an answer. "Why does zero times seven equal zero?" The child is being asked to show awareness of the principle by saying something like "when zero is a factor the product is zero" or "zero times anything equals zero." A written example of a process question would be "7 + 3 = 10 and 3 + 7 = 10, why?" The student is expected to respond with something like "changing the position (order) of the addends (numbers) does not change the sum."

In summary, product questions are those questions that ask students to provide the right answer (how much, what, when). In contrast, process questions ask students to explain how an answer was or could be obtained (why questions).

Problem

Often when teachers think about development and conceptual work, they equate it with process questions. This is *not* the case.

Indeed, often process questions are overused or used inappropriately. The problem with process questions is that they are sometimes ambiguous to the student (what is the teacher asking me?) and may produce an ambiguous student response even though the student understands the concept. Process questions often consume a lot of instructional time (student thinks, mentally practices the response, makes an oral response). Hence, if process questions are overused, a lot of instructional time can be wasted. If selectively used, process questions can be very valuable. For example, by asking a few process questions, teachers can see if students understand the rationale or principle upon which computational work is based and help consolidate student learning.

If teachers are alert to student responses, hold students accountable by asking individual students questions, and keep all students involved in the lesson, then the learning of unproductive habits is minimized. If process and product questions are used appropriately, then student involvement and achievement are enhanced. If they are used inappropriately, then much instructional time is lost and errors are practiced—errors that subsequently are very hard to correct.

Teaching Suggestion

We feel that the presence of a few process questions in the development stage of a lesson is helpful (especially when a new principle is being introduced) because listening to a student's explanation can help teachers diagnose inappropriate assumptions, etc., that students have made. However, we believe that most of the process development can be done through teacher modeling of *process explanations* rather than by asking students to respond to process questions. For example, the teacher could ask,"Who can tell me what zero times seven is?" The teacher surveys the room and calls upon Bill (who may or may not have his hand up). When Bill says "zero," the teacher could respond with something like, "That's right, Bill, the answer is zero. Whenever zero is a factor, the product is always zero." By actively verbalizing and demonstrating (e.g., writing problem solutions on the board), teachers can help students to achieve process understandings in a very efficient way. Still it is useful to ask process questions occasionally to assess student understanding. However, if asked properly, product questions can provide information that assesses the student's ability to relate ideas, transfer concepts to different situations, and understand the process sufficiently well to solve problems. Product questions can *also* provide all students in the class (or group) a chance to practice the computation. This is especially true when the teacher

asks the question first and then calls on a student. If a teacher names a student and then asks the question, many of the students will not perform the calculation (that's Mary's problem). Similarly, if teachers hold nonvolunteers accountable on occasion, it increases the number of students who are likely to think about the problem under discussion.

Although a major goal of the development portion of the lesson is to strengthen students' conceptual understanding (why), this goal can be achieved with a heavy use of product questions. The usefulness of product questions is due to the following factors: (1) they typically elicit a quick response from the student (and quick feedback from the teacher); hence, more material can be covered in a given amount of time; (2) they provide more practice opportunity for a broader number of students; hence, a teacher's diagnosis is not limited to the responses of a few students; and (3) they help to create a "can do" attitude on the part of students (a series of quick questions that the students respond to successfully). However, it is desirable to ask process questions and enter a diagnostic cycle (reteaching) when students respond to product questions incorrectly. When students miss the same type of product questions, then it is useful to stop and review the process and ideas behind the computation. To reiterate, process questions can and do play a valuable role in successful mathematics teaching, although they should not be overused.

APPENDIX B

TYPICAL HOMEWORK ASSIGNMENT

Reproduced below is a page from the fourth-grade Holt mathematics textbook. An appropriate homework assignment would be to assign problems 4 through 18 (evens). The remaining problems could be used in connection with the development or seatwork portions of the lesson. Appendix C shows how these same problems could be put in a different format and thus provide some variety in your assignments.

EXERCISES

Add. Look for patterns.

1.	3	13	23	43	73
	+ 6	+ 6	+ 6	+ 6	+ 6
2.	4	14	24	64	84
	+ 7	+ 7	+ 7	+ 7	+ 7

Add.

3. 41	4. 65	5. 93	6. 14
+ 2	+ 2	+ 2	+ 2
7. 23	8. 41	9. 65	10. 84
+ 8	+ 9	+ 6	+ 9
11. 84	12. 36	13. 48	14. 36
+ 6	+ 9	+ 8	+ 7

Solve these problems.

15. 17 cents for candy.
8 cents for gum.
How much in all?

16. 76 players.
3 more joined.
How many now?

17. 35 pounds of oranges.
9 pounds of apples.
How much fruit?

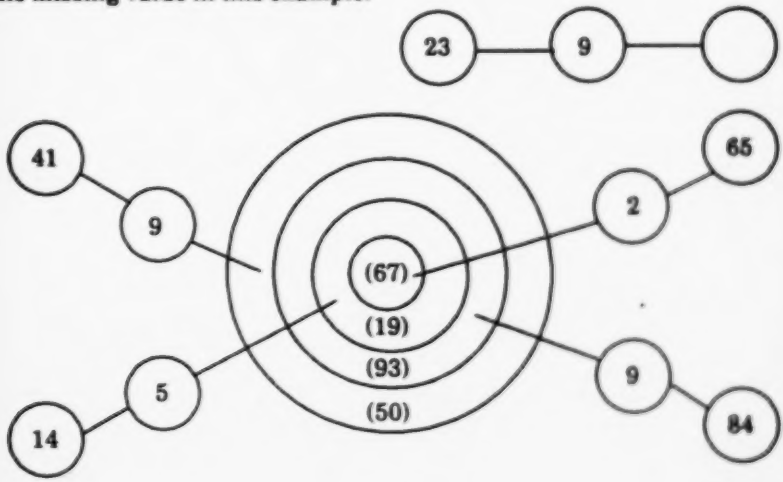
18. 24 bees.
8 ants.
How many insects?

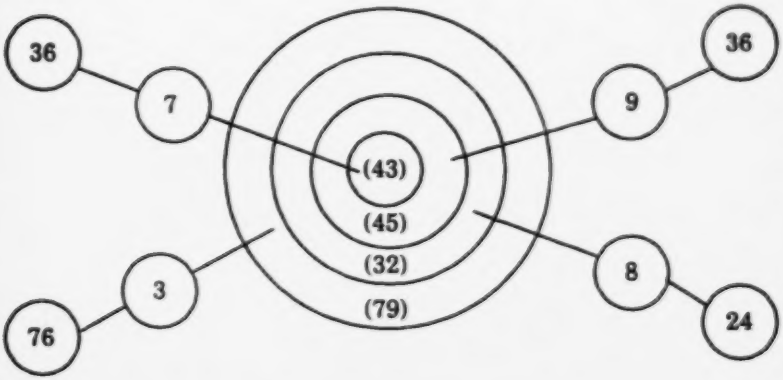
APPENDIX C

VARIETY IN ASSIGNMENTS

Frequently students can be freed from the somewhat boring routine of always doing problems from the textbook as their homework assignment. The assignment shown below is an alternative to the typical row-by-row set of computation exercises found in most textbooks, yet it accomplishes the same objective in a more interesting format. Answers for the problems are shown in parentheses.

Add to find the missing target values. For example, 32 would be the missing value in this example:





APPENDIX D

SYSTEMATIC PROCESSING ERRORS ILLUSTRATIONS

A systematic processing error is an error a student consistently makes on a particular kind of problem. It is different from making random errors. Simple examples include always working addition from left to right or "borrowing" in every subtraction problem whether or not it is necessary. Other common examples are explained below.

In each of the following situations, carefully analyze the examples and try to determine the error pattern. Then check your work by reading the description of the error pattern.

Situation #1

23	34	29	38
+ 6	+ 9	+ 5	+ 4
83	124	79	78

ERROR PATTERN: In these problems the student does not add straight down a column, but rather adds the number of tens from the first number to the units from the second number. Thus, in example #1 the two tens are added to the six ones to get eight tens.

Situation #2

53	86	95	31
- 27	- 39	-27	- 19
34	53	72	28

ERROR PATTERN: In these problems the student does not "borrow" but rather always subtracts the smaller digit from the larger digit.

Situation #3

7	5	3	5
48	49	86	67
x 59	x 36	x 45	x 28
432	294	430	536
270	177	354	174
3132	2064	3970	2276

ERROR PATTERN: The first part of each problem, the multiplying by the ones, is done correctly. However, when multiplying by the tens, the crutch number recorded from the multiplying by ones is incorrectly used again. For instance, in the first example, when multiplying by the 5 tens the 7 (carried over from the 9 x 8) is used again when the 7 is added to 5 times 4 and the 27 is recorded.

Situation #4

4	1	3	2
26	83	38	53
x 7	x 5	x 4	x 8
422	455	242	564

ERROR PATTERN: In these problems the crutch is added before multiplying in the tens place, whereas the correct procedure is to multiply and then add the crutch. Thus, in the first example the 4 is added to the 2 and then this sum is multiplied by 7. If this problem was done correctly, the 2 is multiplied by the 7 and then the 4 is added.

Situation #5

44	14	87	39
2/88	4/164	3/234	5/465
80	160	210	450
8	4	24	15
8	4	24	15

ERROR PATTERN: These problems are worked correctly except that the quotient figures are written from right to left. Consider the third example, there are 7 threes in 23, but the 7 is recorded at the extreme right, rather than above the 3.

Situation #6

```
      32r3
   ------
9 ) 2721
    27
    ---
     21
     18
    ---
      3
```

```
      78r2
   ------
6 ) 4250
    42
    ---
     50
     48
    ---
      2
```

```
      94r2
   ------
3 ) 5426
    54
    ---
     26
     24
    ---
      2
```

ERROR PATTERN: In these problems whenever the student brings down and cannot divide, he brings down again but forgets to record a zero in the quotient.

APPENDIX E

LETTER TO PARENTS

September 7, 1983

Dear Parent(s):

As part of the fourth-grade mathematics instructional program this year, I will be regularly assigning some work for the students to complete at home. It should take your son or daughter about 15 minutes to complete this homework. If you find that it regularly takes considerably longer for him/her to finish this assignment or the assignment causes other difficulties, please let me know in that I may be assigning too many or too difficult problems.

Programs in other school districts, educational research, and common sense indicate that the more students practice important mathematics concepts and problems, the more proficient they become in essential mathematics skills. I view homework as an opportunity for the student to practice the concepts and skills that he/she has learned in class. I hope that you will encourage your son or daughter to complete every assignment to the best of his/her ability. Parental support is very helpful. Thank you for your cooperation in this matter.

Sincerely,

APPENDIX F

TEACHING GROUPS IN SCHOOLS USING DEPARTMENTAL ORGANIZATION

The emphasis thus far has been placed upon teaching mathematics to the class as a unit. We feel that many of the principles presented (the importance of development, the use of controlled practice and seatwork, accountability, etc.) will transfer to classrooms in which teachers are teaching groups of students. In applying these principles to a group situation, teachers will have to *adjust* them to their teaching situation.

In general, we are not enthusiastic about the use of two or more groups to teach mathematics. Three recent and major research projects have shown that third-, fourth-, and fifth-grade students appear to benefit more from whole class instruction than they do from individual or group instruction. Although the precise reasons for these differences are unknown, we suspect that students learn less in group and individual settings because they have less direct developmental work with the teacher. Also, the extra transition (teachers moving from group to group) probably results in the loss of time that could have been used for instructional purposes. Furthermore, student work is probably less effective when the teacher is not available to supervise work.

If the differences between groups are not great, we strongly recommend that the class be taught as a whole class. However, we understand that sometimes the differences between students in a given classroom are so great that grouping is a practical *necessity*.

If grouping is necessary, you should attempt to limit yourself to only *two* groups because the transition and supervision problems that accompany the use of more than two groups are normally very difficult to justify.

Since teaching circumstances are so varied (sometimes the difference between two groups is moderate, but in other classrooms there are vast differences between the two groups), it is impossible for us to describe a plan that would be best in all situations. Still, there are a few key things that we would like to emphasize.

First, whenever possible we think it will be useful for you to teach the class as a group. Students learn a great deal from teacher illustrations and explanations. Perhaps the easiest way to do this in a group situation is by holding common reviews from time to time. The review might be a short-term review for the lowest group and a long-term review for the highest group.

An especially good way to conduct a common review is through the use of mental computation problems. We strongly recommend that each day of the week but Monday you use the first *ten minutes* of the class for review with mental computation problems. As we have noted earlier in the chapter, we feel that mental computation problems are a very important addition to an instructional program.

Second, we suggest you set aside each Monday for a review session. After spending the first five minutes on mental computation, review ideas and skills that are needed by both groups. Involve one group in a seatwork review, then begin the developmental review with the other group. Roughly halfway through the period, reverse the roles; give group two a seatwork review assignment and begin an oral review with group one.

To maximize the value of this review, a homework assignment containing review problems should be given the previous Thursday. Your analysis of these papers should suggest the topics and skills that should receive emphasis in the Monday review. Besides the homework assignment each Thursday, we suggest that you assign homework three other days per week. Remember that these assignments are to provide *brief, successful practice*.

The third suggestion is that you maximize the amount of development time for each group. The exact amount to be given to each group will necessarily vary depending on the topic being considered and the group itself; however, the importance of development work for both groups cannot be overemphasized. As you do the development work, remember the guidelines previously discussed. For instance, teacher explanations and illustrations are important, especially initially. Also, process explanations are very important and oftentimes are related to efficient use of limited instructional time.

Finally, we suggest that you implement other recommendations as regularly and consistently as you can. Little things are important (e.g., getting all students started on seatwork before doing other instructional tasks), and we hope you will carefully review the ideas presented in this chapter with an eye toward applying them in your classroom.

CHAPTER 4

EFFECTIVE MATHEMATICS TEACHING IN THE JUNIOR HIGH SCHOOLS*

by

Thomas L. Good and Douglas A. Grouws
University of Missouri

Introduction

This chapter describes modification of procedures identified in Chapter 1 for junior high school classes. The modifications include substantive changes as well as minor adjustments. The changes resulted from a group meeting of junior high mathematics teachers who read the materials and then met to discuss the program. The following revisions reflect the collective thinking of the group.

Daily Review

During the introductory phase of the lesson, a number of things must take place: a brief review, checking homework, and some mental computation. There was agreement that teachers should move rapidly through this phase, because there is a tendency to spend too much time going over homework, and also because the review at this point is distinct from prerequisite skills in the development portion of the lesson. However, in light of the additional mathematics time available at the junior high school level, it was decided to extend the time allotted for the introductory phase to 12 minutes, as shown on the appended Junior High Program Description.

Development

The ten minutes following the introductory activities are designated for instruction in verbal problem solving, using strategies outlined in Chapter 5, Verbal Problem Solving. The ability to compute in

*The development of this material was supported by Grant NIE G 79 0103 from the National Institute of Education. It is reprinted here with permission from the authors.

isolation is of very little value unless students can use the skills in a variety of situations. It should be emphasized that the time devoted to verbal problem solving should involve the teacher teaching and the students participating in class discussion. This recommendation is *not* fulfilled by just having the students work a few word problems at their desks. Teachers need to be prepared to teach in order to follow this program and this means having the verbal problems to be used in this part of the lesson determined before the period begins. There is often a variety of textbooks available at school, and these can be a very useful source of problems for this part of the lesson.

Active, careful, and meaningful teaching of the topic for the day in the development portion of the lesson is important. A smooth transition from the development phase to the seatwork phase (where students work individually on a collection of problems or exercises at their desks) is necessary. Much time can be lost if this transition is not carefully managed. The program suggests that teachers control practice in the latter part of the development phase; that is, teachers should have students work a problem like the first problem in their assignment and then discuss and demonstrate its solution in front of the class. This procedure should be repeated several times until all the students seem to have the idea. This controlled practice helps a great deal in getting students started immediately after the seatwork assignment is given.

Seatwork

The procedure for seatwork as described in Chapter 3 is not changed. Teachers are to make sure that each student is working before providing individual help. During seatwork, the teacher should circulate about the room, supervising student work to assure that students are not practicing incorrect methods.

Holding students accountable at the end of the period can be done in several ways, including: (1) calling on some students to give their answers to particular problems; (2) checking the answers to the first few problems orally (and then occasionally taking grades); and (3) calling on students and asking how many problems they have finished. Other methods are also possible. The important thing is that students do some problems in class while the skills and ideas are fresh in their minds and while help is available for those who need it.

A homework assignment on Monday through Thursday is a reasonable requirement, and students this age can be expected to work on homework for longer periods of time than those specified in Chapter 3. It was decided that assignments should take 15 to 30 min-

utes. This flexibility allows for the different rates at which individual students work and also permits teachers who like to give a combined seatwork-homework assignment to do so. That is, a given number of problems is assigned and students are allowed to work on them during the 10 to 20 minutes of seatwork in class; whatever problems are not finished are completed outside of class as homework. Under these circumstances, the assignment should be long enough to provide some homework for everyone.

Several teachers mentioned that junior high students need to assume more responsibility than they have previously and that teachers should thus expect homework to be done on time. Some teachers suggested different ways of recording homework grades. (For example, on some days they score the papers and record the grades, and on other days they just check off complete papers. In this way students do not know which procedure is to be used on a particular day.) The way teachers handle collection of homework is left to the discretion of individual teachers.

The discussion of the structured reviews every Monday centered on how they should be conducted and the flexibility in scheduling them. There was agreement that the review need not be solely lecture but should include student interaction in the form of student questions, having students go to the board, or having students work a review problem at their desks with discussion following. The possibility of a weekly quiz during *part* of the review time was discussed, and this was deemed acceptable.

The timing of quizzes and tests was discussed, and this scheduling was left to the individual teacher.

Finally, the topic of occasional variation from the time framework was discussed, and the consensus reached was that teachers should adhere as closely as possible to the guidelines, realizing that some variance from the guidelines may be unavoidable. However, in such cases, each teacher should make a concerted effort to adhere as closely as possible to the guidelines.

JUNIOR HIGH PROGRAM DESCRIPTION

The treatment involves the following:

1. The beginning portion of the lesson (first 12 minutes) should have three parts:

a. A brief review
b. Checking of homework
c. Three to five minutes of mental computation exercises

2. Approximately 10 minutes daily should be spent on verbal problem solving.

3. Approximately 20 minutes daily should be devoted to developing conceptual understanding of mathematics.

4. Approximately 30 minutes of homework should be assigned each night except Friday.

5. Approximately 10 to 15 minutes of seatwork should be given daily to provide uninterrupted successful practice.

6. The first 20 minutes each Monday should be spent on review and the full period used for review every fourth Monday.

7. The teacher should get everyone involved in the seatwork immediately and keep them involved.

8. The teacher should make the students accountable for the seatwork.

JUNIOR HIGH TIME ALLOCATIONS

Daily Schedule for Tuesday through Friday

Phase 1: Introduction should include:

1. One to two minutes on review of concepts and skills associated with the homework

2. Three to five minutes spent checking, collecting, and dealing with homework assignments

3. Two to five minutes spent on mental computation

Phase 2: Approximately 28 minutes should be devoted to the development of the mathematics of the lesson. This phase should include four parts:

1. Verbal problem solving (10 minutes)
2. Comprehension phase
3. Teacher questions to assess comprehension
4. Controlled practice

Phase 3: Approximately 15 minutes should be spent on seatwork.

Phase 4: Assign homework (Monday through Thursday).

Schedule for Monday

Phase 1: The first one-half of each Monday's class period (about 25 minutes) should be devoted to review/maintenance.

Phase 2: Approximately 20 minutes on development

1. Verbal problem solving (10 minutes)
2. Comprehension phase
3. Teacher questions to assess comprehension
4. Controlled practice

Phase 3: Approximately 10 minutes of seatwork

Phase 4: Assign homework.

Schedule for Every Fourth Monday

The entire mathematics period should be devoted to a cumulative review/maintenance session every fourth Monday.

CHAPTER 5

VERBAL PROBLEM SOLVING*

by

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Introduction

There are many reasons for teaching students mathematics, and different people stress different reasons as they testify to its importance. On one thing, however, there is universal agreement: mathematical problem solving is of paramount importance! This agreement stems from the fact that many real world problems are most easily solved by expressing and treating them mathematically. An important step toward developing problem-solving ability in students is to help them gain competence in solving verbal problems. By verbal problems we mean those problems which are commonly referred to as "story problems" or "word problems." These are the problems that are traditionally found in contemporary student mathematics textbooks.

In the past, instruction on verbal problem solving has amounted to little more than the teacher solving a few sample problems in front of the class and then asking students to solve similar problems on their own. Usually such instruction is grossly inadequate; students do not understand the assignment and are not able to do the problems successfully. Because of such poor presentation, many students develop a permanent dislike for these problems. This situation is particularly unfortunate because research has shown that there are a number of instructional strategies that can be used to improve student problem-solving performance significantly. The remainder of this chapter is devoted to describing techniques that can be incorporated successfully into daily instructional practice. When these techniques are used systematically, we believe that students' ability to solve verbal problems will show steady progress.

In particular, it is important to include some work on verbal problem solving each day. Too often verbal problem solving is taught

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only three or four times a year as a special topic. However, it is only the day-to-day, brief but systematic exposure that will allow students to become proficient in solving mathematical problems.

PROBLEMS WITHOUT NUMBERS

The use of problems without numbers is a very effective instructional technique for improving verbal problem-solving performance. It provides students an opportunity to gain insight into the problem-solving process by avoiding the use of numbers and thus the need to perform any computation whatsoever.

Example

To illustrate the method, consider the following typical problem:

Two classes sold 100 football game tickets.
One class sold 27 tickets.
How many did the other class sell?
(*Holt School Mathematics, Grade 6*, p. 32)

This problem can easily be rephrased so that it is a problem without numbers:

Our class and Mrs. Smith's class sold tickets.
We know how many tickets were sold altogether and how many tickets our class sold.
How many tickets did Mrs. Smith's class sell?

The teacher presents only the problem without numbers and asks the class *how* to solve it. An appropriate answer might be something like this: "I'd subtract how many tickets we sold from the total number of tickets to find how many tickets Mrs. Smith's class sold." Time permitting, the teacher should follow-up with another problem without numbers or occasionally consider the same problem with the numbers included.

Rationale

The specific reasons why this technique is effective are difficult to isolate. One reason for its effectiveness may be that it causes students to focus exclusively on the *method* needed to solve a problem without any numerical or computational distractions. Many teachers realize

that too frequently students begin doing the computation before they have really thought through the problem. In fact, some students have been known to begin computing before they have read the entire problem! Avoiding the use of numbers tends to resolve these kinds of problems. Since the strategy does not require computation, students can be exposed to a substantial number and variety of verbal problems in a short period of time.

Implementation

This technique should be used frequently as part of a comprehensive effort to improve verbal problem-solving skills. It seems especially effective if teachers create the problems to be used by recasting verbal problems found in the student textbook. It is also helpful if the problems are written down and ready for presentation prior to the beginning of mathematics period. This allows efficient use of the available instructional time.

WRITING VERBAL PROBLEMS

Research has shown that when students create and write verbal problems, their problem-solving ability improves. Certainly a comprehension of what constitutes a problem is necessary in order to succeed at writing problems, and this in turn may be a vital component in learning to solve verbal problems.

Example

There is a variety of interesting formats that a teacher may use when having students write verbal problems. One method is to supply data and ask students to make up their own problems based on this information. For example, the data might consist of a football team roster like the one below.

Number	Player	Position	Year	Height	Weight
11	Anderson, Bill	Quarterback	9th	5'8"	155
24	Baker, Burt	End	8th	5'7"	140
17	Brunson, Jim	Quarterback	8th	5'5"	135
..	.....	.....	...	...	...
..	.....	.....	...	...	...

To illustrate what kinds of problems may be written, the teacher could supply examples like the following which range from easy to complex:

Bill Anderson and Jim Brunson are both quarterbacks on the Memorial Junior High School football team.
Bill weighs 155 pounds, and Jim weighs 135 pounds.
Bill weighs how much more than Jim?

There are three quarterbacks on the Memorial team.
Jim weighs 135, Bill 155, and Sam 130.
What is the average weight of the quarterbacks?

All 33 players on the Memorial team are going on the bus to the away game with Fulton Junior High.
Highway 24 is the shortest way to Fulton, but the Mason Creek bridge on this route limits loads to less than five tons.
The bus with the driver weighs 3,200 pounds.
Will the bus loaded with the players be too heavy to use Highway 24?

After students have had some experience writing problems, the teacher may allow them to make up problems by supplying their own data from situations that are of interest to them. Placing some restrictions on the problems to be written will help to keep this activity consistent with the operations and kinds of numbers currently being studied. For example, a teacher might want to restrict the problems written to those that can be solved by division of whole numbers or to those involving addition and subtraction of fractions.

Rationale

The value of having students create verbal problems is closely tied to their simultaneous development of the ideas of information given, information to find, and a link or path from the former to the latter. Writing a problem requires attention to all three components. In the early stages of this development a student may only consider the given aspect and write a "problem" like:

Suzi has 9 packages of baseball cards.
There are 12 cards in each package.

As students progress in their ability to comprehend what constitutes a problem and thus the ability to write problems, there is likely to be some transfer to those situations where students are presented with problems to be solved. This transfer may be in the form of recognizing what is given, what is to be found, or that the task is to build a bridge or link between the two. The importance of this transfer is emphasized by the number of times we have all heard the comment: "I really don't know where to begin." If teachers regularly have students write verbal problems, they should hear this question much less frequently.

Implementation

This technique can be closely tied to instruction on any of the basic operations (addition, subtraction, multiplication, and division) as well as most other topics, including measurement and geometry. Students may be asked to write problems in class, as part of a homework assignment, or both.

Allowing students to solve one anothers' problems often stimulates their interest. Contests based on ideas like "stump the teacher" and "problem of the week" also add variety and interest.

ESTIMATING THE ANSWER

Students who estimate the answers to verbal problems before they attempt to solve them seem to make important gains in their ability to correctly solve problems. Use of this technique is not difficult, yet the payoff from using it can be substantial.

Example

Students can be asked to estimate the answer to any verbal problem. Consider this problem:

> Janet picked 17 daisies for each of her classmates.
> She had 38 classmates.
> How many daisies did she pick in all?
> (*Holt School Mathematics, Grade 6*, p. 63)

Students may estimate the answer to be 600 by formally thinking of the product 15 x 40, or by informally thinking of 15 sets of 40. Another estimate might be 700 by thinking that the answer will be somewhat less than 20 x 40. Each of these estimates is close enough to

the exact answer of 646 to serve the desired purpose. Of course, students may estimate the answer in an entirely appropriate way that is very different from the formal and informal methods mentioned here. A discussion of the methods used to estimate a particular answer can be very enlightening for students and teachers alike. In particular, such discussions provide an excellent learning experience for those students who have a poor concept of what is involved in the estimation process.

Rationale

The benefits derived from using the estimation strategy may be due to several factors. In order to estimate the answer to a problem, a student must comprehend, at least in an intuitive way, what the problem is about. This is an important first step in solving a problem. A reasonable estimate of the solution also suggests and eliminates certain computational procedures. For instance, in the previously cited example, the operations of addition, subtraction, and division are ruled out quickly since there is no way they can operate on the numbers in the problem (17 and 38) so that the result will be anywhere close to a reasonable estimate; in fact, such operations would not even yield a three-digit number!

Another factor which may contribute to the value of estimation is that it provides a safeguard from absurd answers and thus provides a means of detecting computation errors. Although there may be other reasons why the estimation technique is so effective, suffice it to say that the results are generally very positive.

Implementation

The estimation technique is easy to use and should be used in two distinct situations. First, it should be used regularly as an instructional method, perhaps by being a part of a regular rotation among other problem-solving methods. Second, once students are acquainted with the idea, they should be required to make and record an estimate of the answer for every verbal problem they solve. Teachers are responsible for soliciting and discussing estimates for all problems worked orally in class. They should also monitor seatwork and homework to insure that students are estimating answers in these situations, too.

One successful approach to monitoring is to have students record their estimates and then identify them by underlining them. Exact answers are then either circled or underlined twice.

It is important to emphasize again that discussion of the various methods of making an estimate for a specific problem is an ideal learning situation for those students having difficulty with this technique. Teachers can also foster the initial development of this ability by thinking aloud as they make their estimates as part of work done in front of the class. Providing practice in rounding numbers and doing mental computation is also beneficial. A teacher must emphasize that in order for an estimate to be helpful it must be carefully made and not a "wild guess." Teachers can best do this early in the year by frequently modeling (thinking out loud) and clearly demonstrating to students how to make estimates.

One final thought to keep in mind as you do estimation work is that estimating can be informal in nature and need not rely on formal calculation, either written or mental. Recall that the product of 20 and 40 can be thought of informally as 20 groups of 40, and the approximate result gained from relying on one's quantitative sense is usually accurate enough to serve the desired purposes outlined in this section.

READING VERBAL PROBLEMS

The inability to read verbal problems is a definite factor in the difficulty many students have in learning to solve verbal problems. Thus, a sustained effort to overcome reading problems is necessary in order to improve verbal problem-solving ability significantly.

Example

There are many facets to the reading process that must be taken into account in the instructional process. To read well, a student must be able not only to "string words together," but also to comprehend these words. Consider this problem:

> The Great Pyramid was originally 481 feet tall.
> The Great Pyramid was as tall as a building of how many stories, if you used 12 feet per story?
> (Addison Wesley, *Investigating School Mathematics, Grade 6*, p. 141)

There are many kinds of reading-related difficulties associated with verbal problem solving. An initial difficulty in the example problem might be with recognition of words like "Pyramid" and "building." Another difficulty, associated with a higher level of thinking, might

be recognizing a word but not associating it with its appropriate meaning. In the example problem, a student might incorrectly think of the word "stories" as being a collection of narratives rather than a measure of the height of a building. Finally, even if the words and their meanings are correctly discerned, there is sometimes difficulty with general comprehension. Among other things, the student must realize what information is given and what is to be determined.

Rationale

A student who cannot read a problem is going to have great difficulty solving it. We now examine a method for handling these reading-related problems.

Implementation

There are two goals to be worked on jointly. First, assistance must be given to students to help them overcome their reading problems. Progress on this goal is oriented toward a long-term solution to the problems, which in turn will result in better problem solvers. The second goal is to provide practice in solving verbal problems which circumvents reading difficulties. This is done by the teacher reading problems aloud, using tape recorders, and so on. The second goal insures that improvement in verbal problem solving will not have to wait until the reading difficulties are remediated, which in many cases may involve a considerable period of time.

Several things must be done as part of our *regular mathematics instruction*, regardless of the particular topic being studied in order to reduce the possibility of later reading difficulties. Terminology must be given special attention. Whenever a new term is introduced it must be written on the board, carefully pronounced first by the teacher and then by the students, and then its meaning must be carefully discussed. This discussion should include both examples and nonexamples of the concept and also distinguish between the mathematical meaning of the word and any nonmathematical use of the word. For example, the word "plane" has a special mathematical meaning quite different from everyday use, in which it might designate an airplane or a hand tool.

Whenever verbal problem solving is the *main topic for a lesson* the teacher must take direct steps to deal with reading problems. This means that all problems presented in the development part of the lesson and the first problem in any seatwork assignment must be carefully read aloud by the teacher or a student and important words

and ideas discussed. An example of how this is done is described later in this section. Students must also be given reading assistance on more than the first seatwork problem. A teacher could effectively make use of audio recordings of the problem or provide reading assistance as needed and requested during the seatwork time.

Special attention to reading problems alone should be included periodically during the daily portion of the mathematics period which is devoted to problem solving. This may involve teachers and students alternately reading problems, with a discussion of each problem after it is read. For example, in the problem:

> Waves as high as 112 feet have been reported on the "high seas." If each floor of a building is 14 feet tall, the wave would be as tall as a building with how many floors?
> (Addison Wesley, *Investigating School Mathematics, Grade 6*, p. 109)

Several meanings of the word "wave" could be discussed, and attention would also be given to identifying the two pieces of information given and what needs to be determined to solve the problem. Problems to be read may be collected from the textbook, teacher and student written problems, and problems from older textbooks which are no longer in use in the school district. In order to focus primarily on reading, especially reading for meaning, problems read and discussed need not be solved. This allows for many problems to be considered in a short period of time.

Progress on reading difficulties should result from the above mentioned suggestions. Of course, progress can also be expected from students due to their regular reading instructional program. Certainly it is quite appropriate for mathematical material to be used as part of this instruction. Finally, not all students will benefit to the same degree from the attention to reading problems, but it will be a valuable experience for some students.

Anyone who has taught verbal problem solving is aware that reading problems which hinder verbal problem solving do not appear in isolation. How many times have you read a problem to a nonreader and he still could not solve the problem? For this reason, attention to reading is only one of the many important techniques that must be given regular attention in instruction on problem solving.

WRITING AN OPEN SENTENCE

Many potential benefits of mathematics learning are realized when mathematics is used to model physical situations, because it is in this way that mathematics is used to solve everyday problems. The simplest situation where this takes place is where an open sentence is written to represent a verbal problem involving a minimal number of conditions. Verbal problems can often be solved without going through this step, but some research has shown that developing the ability to translate problem conditions into mathematical sentences is related to improved problem-solving performance.

Example

To illustrate how an open sentence can be used to model a verbal problem of a real world situation, consider this example:

> Nine classes in the school gave a total of $1,080 to the Book Fund. Each class gave the same amount. How much did they each give?
> (*Holt School Mathematics, Grade 6*, p. 87)

This problem can be translated into the open sentence 9 x ____ = 1,080. The answer to the problem is then found by solving the open sentence using informal means such as estimation or by formal calculation of the quotient 1,080 divided by 9.

Rationale

As with many other successful techniques, this technique probably gains much of its power by forcing the student to read carefully and to come to grips with the meaning of the problem. That is, it is necessary to determine how the given information pieces relate to one another in order to write an appropriate open sentence. Another reason for the usefulness of this method is that it reduces memory load in complex situations. Given a complex problem involving many conditions, it is difficult, if not impossible, to mentally remember, manipulate, compare, and contrast the given conditions. If, on the other hand, these conditions are represented in the form of a collection of open sentences, the task becomes much more manageable. For example, the following problem is difficult to solve without the use of open sentences.

In order to put a fence around two adjacent sides of a rectangular lot, 38 feet of wire is needed. The area of the lot is 217 square feet. What is the length of each side of the rectangle?

Let x and y represent the length of the two adjacent sides. Then $x + y = 38$ and $xy = 217$. From the first sentence we know $y = 38 - x$ and when we substitute this into the second sentence, we have $x(38 - x) = 217$. This can now be solved by trial and error or more formal means, but in either case getting the problem into manageable form involved writing open sentences (equations).

Implementation

This problem-solving method should be taught to all students. Hopefully most students will already have had prior exposure to the technique, and thus only periodic review will be necessary in order for them to use the technique as they solve verbal problems. The periodic review can be part of a rotation among other techniques described in this chapter and like the others can be done daily using a small part of the mathematics class period.

When providing practice on translating verbal problems to open sentences, several important ideas need to be remembered. First, there will be a tendency for students not to write open sentences for the very simple problems. The typical comment will be "I already know how to do it!" The teacher must persevere and require that sentences be written in most cases, because only in this way will the skill be learned and the student develop the capability to apply the skill in more complex situations. Attempts to translate difficult problems involving many conditions to open sentences without considerable practice on simple problems usually end in failure.

Second, teachers must be aware that several different open sentences may equally well model the same verbal problem. In the example already described the sentence $1{,}080 \div 9 =$ _____ could be used as a model quite appropriately. It's likely, however, that many students will think of the problem multiplicatively and write the sentence $9 \times$ _____ $= 1{,}080$. Either sentence is acceptable and both lead to a correct solution. Finally, make students aware that any one open sentence may model a large number of situations that seem perceptually different but are alike structurally.

Several other suggestions may be of help. When this technique is being used as just a small segment of the lesson (e.g., ten minutes), it will frequently be desirable to have students write only the open sen-

tence that goes with a problem and not continue to find the exact solution. Also, for those students who have had little work with using open sentences in this way, it will be easier if the initial translations involve simple problems. Using problems from textbooks at lower grade levels is often a good idea under these circumstances.

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